An Equivalent Electric Field Source for Wideband FDTD Simulations of Waveguide Discontinuities

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***Abstract*—Transparent sources are often used in wideband finite-difference time-domain (FDTD) simulations of waveguide discontinuity problems. Direct implementations of transparent sources may not correspond to the original hard source spectrum, or else require auxiliary simulation steps for correction purposes. In this letter, we propose a more direct approach to construct FDTD transparent electric field source—models for waveguides. We also provide an explanation for a "spectrum distortion" phenomenon recently reported for FDTD sources in waveguide problems [9]. The main findings are illustrated by means of FDTD simulations of waveguides operating at Ku-band.**

***Index Terms*—FDTD methods, source, waveguide.**

I. Introduction

THE finite-difference time-domain (FDTD) method [1] has been widely used for wideband waveguide discontinuity simulations. In order to increase the efficiency and accuracy of the method, various source excitation schemes to couple energy into the FDTD grid have been proposed. For waveguide problems, these can be roughly classified into four basic types: 1) hard sources, 2) total field/scattered field (TF/SF) sources, 3) matched modal source boundary condition, and 4) transparent sources.

Hard sources [2], [3] are implemented by enforcing a particular value for the transverse field at excitation points in the FDTD grid. For an incoming wave, these grid points behave as scatterers (e.g., a PEC wall [4]) and, therefore, enough buffer space must be placed between the excitation region and any waveguide discontinuities to prevent reflections from contaminating the solution. TF/SF sources [5] employ the same approach as the homonymous sources used in scattering problems. In this case, an auxiliary simulation step of an empty waveguide is needed to obtain the incident transverse magnetic and electric fields separated by half a cell. Matched modal source boundary conditions [6], [7] introduces the source and an absorbing boundary at the same location. To numerically exactly absorb one or more modes, an auxiliary simulation step is again needed to obtain the impulse response of the waveguide (analytical solutions do not work well [7]). Transparent source excitations [8] modifies the field update equations by introducing source terms. Since there is no interaction between the source and the reflected wave from the discontinuities, transparent sources are often the most adequate type of FDTD excitations for waveguide discontinuity problems. However, if proper care is not exercised on how to define the transparent sources, the resulting fields coupled to the FDTD grid may not correspond to fields of an equivalent hard source [4]. In order to construct a transparent source which couples into the FDTD grid the same field as a hard source, Schnider *et al.* [4] have proposed a pre-simulation step to obtain the impulse response of the source PEC wall. During the actual simulation, a convolution is then applied at each time step to make the transparent source yield the same spectrum as the equivalent hard source.

Recently, there have also been reports [9] that the use of some FDTD source excitations for waveguide problems may lead to large "spectrum distortions" in the field spectrum at the observation point. These so-called distortions produce a large peak in the spectral energy near the cutoff frequency.

In this letter, we consider a simple transparent source excitation approach for waveguide problems which avoids any convolutions and couples the same field (and hence the same spectrum) as a hard source. We shall also provide a simple explanation for the "spectrum distortion" phenomenon as reported in [9]. For simplicity, a dominant TE_{10} mode in an rectangular waveguide will be considered as an example.

II. Equivalent Current/Field Source Models

Considering a waveguide whose broadside extends along the y-axis direction and the z-axis direction is the propagation direction, the spatial distribution for the source with a TE_{10} mode can be expressed as

$$\mathcal{S}_x(j_{src}) = \sin\left(\frac{\pi j_{src}\Delta h}{a}\right) \quad (1)$$

where a is the broadside dimension, Δh is the cell size, and j_{src} indicates the spatial location along the y-axis. At this point, we choose to be deliberately ambiguous as to which particular type of source (i.e., current or field, magnetic or electric) this term would represent. First we consider the electric field update equations with the above source included as in [8]

$$E_x^{n+1}(j_{src}) = E_x^n(j_{src}) + \frac{\Delta t}{\epsilon}\nabla\times\vec{H}^{n+\frac{1}{2}} + S_x(j_{src})\cdot p^{n+1} \quad (2)$$

where the superscripts denote the time step and p^{n+1} is the time domain pulse evaluated at time step $(n+1)$.

To examine the effect of such an excitation, we simulate a TE_{z10} wave in a Ku-band waveguide (WR62: 15.799×7.899 mm^2). A uniform grid with cell size

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$\Delta h = 0.6522$ mm is used. The cutoff frequency of the waveguide is around 9.494 GHz. A differentiated Gaussian pulse is used with an frequency range of 12–20 GHz. The observation point is located 10 cells away from the source plane and a mode extraction technique [6] is used to extract the modal voltages [7]. The first 300 time steps of the FDTD simulation are shown in Fig. 1 and the frequency domain results obtained by a discrete Fourier transform (DFT) using 1700 time steps are shown in Fig. 2 (denoted as *FDTD H source*). Theoretical results in the time domain are also generated by inverse DFT from the theoretical results in the frequency domain and denoted as *Theoretical* in these figures. As we see, large oscillations occur in time domain and the spectrum has a peak near the cutoff frequency (somewhat smeared out in Fig. 2 because of the limited resolution of the DFT).

The key to treat this problem is to understand the effect of the source S_x. Because of the way it couples into the electric field update, it can not be treated as an electrical field source, as done in [8]. Instead, it must be treated as a current source, cf.[4]. In this case, (2) effectively adds an electrical current J_x, which does not give rise to TE_z waves. Consequently, the modal voltage depicted in Fig. 1 is understandably different from the theoretical TE_z result. Analogously, one should not conclude that by adding a magnetic current source M_z, a TE_z wave would be excited. The scaled time domain result for such a source is also included in Fig. 1 and denoted as *FDTD M source*, where large oscillations are also present.

The peak on the *FDTD H source* curve in Fig. 2 is due to the fact that this figure actually refers to the electric field produced by a magnetic field source. As such, the wave impedance of the TE mode in a waveguide needs to be properly taken into account [10]. Near the cutoff frequency, the waveguide acts as a strongly dispersive system and the TE wave impedance actually exhibits a singularity at the cut-off frequency. It is this singularity which causes the oscillations shown in Fig. 1 and the phenomenon reported in [9].

The multiplication of the theoretical spectrum by the wave impedance yields the result denoted as *Theoretically modified* in Fig. 2. This result closely resembles the spectrum of the observation field E_x (the discrepancy is caused by the limited resolution of the DFT).

III. Equivalent Electrical Field Source

In order to excite the structure with the correct frequency spectrum for the electric field, care must be exercised in choosing the proper excitation. Note that the correct location to add a magnetic field source would be (2), where we can assume an equivalent magnetic field H' such that

$$\nabla \times \vec{H}'^{n+\frac{1}{2}} = S_x(j_{src}) \tag{3}$$

is added at each time step. Similarly, the H_y update equations would be the correct location to add an equivalent electric field source for E_x in this case. For a source located on a normal cross-section of the waveguide, this can also be understood by invoking image theory (the field being periodically mirrored along the two directions of the cross-sectional plane) and the

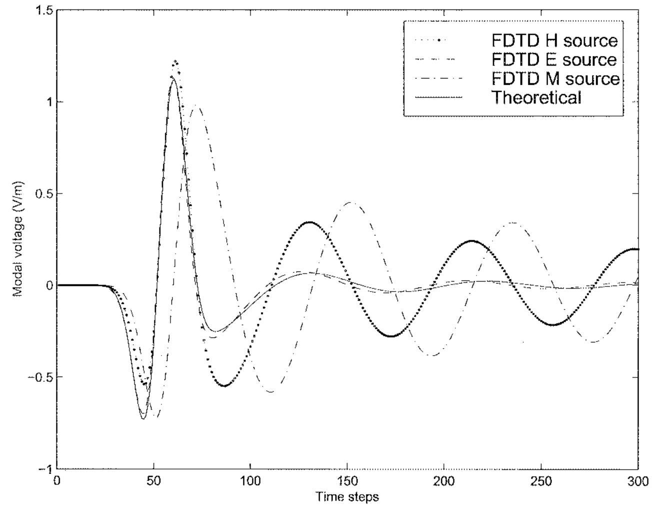

Fig. 1. Comparison of time domain results.

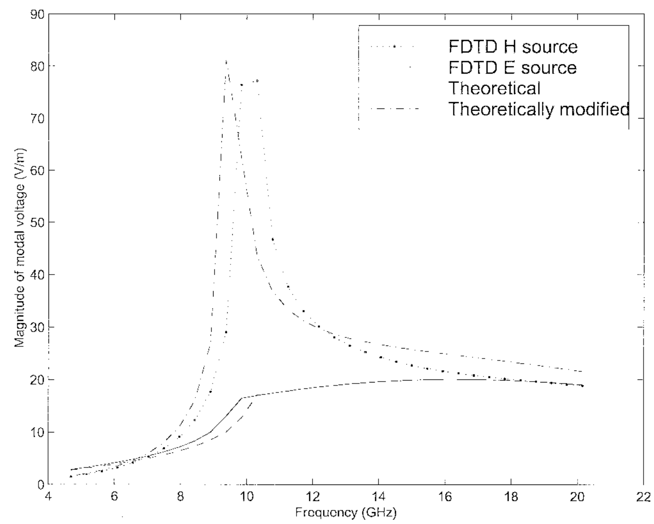

Fig. 2. Comparison of frequency domain results.

free-space equivalence principle, whereby the tangential electric field source E_x becomes equivalent to a tangential magnetic current source M_y [10]. This yields the following H_y update at the source location

$$H_y^{n+\frac{1}{2}}(j_{src}) = H_y^{n-\frac{1}{2}}(j_{src}) - \frac{\Delta t}{\mu}\nabla \times \vec{E}^n + S_x(j_{src})\cdot p^n. \tag{4}$$

Because we add an equivalent *electric* field and observe the *electric* field, the correct spectrum is observed. This is again illustrated by Fig. 1 and Fig. 2, where the time domain FDTD simulation result by applying such a transparent source and its frequency response are denoted as *FDTD E source*. The small discrepancies from the theoretical prediction is mainly caused by numerical dispersion. In the frequency domain, the numerical dispersion and DFT finite resolution together cause the cutoff frequency to shift toward high frequencies by a small amount.

As noted in [4], for applications where one is interested only in the spectral response of a device (e.g., determination of S parameters) and nonlinear materials are not present, the particular spectrum of the source is often not of primary importance (as

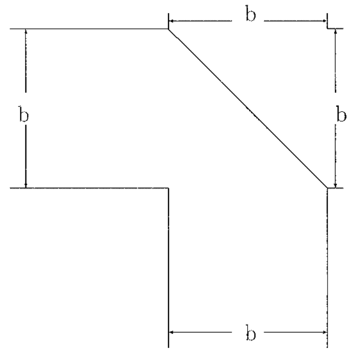

Fig. 3. Geometry of the mitered bend, $b = 7.899$ mm.

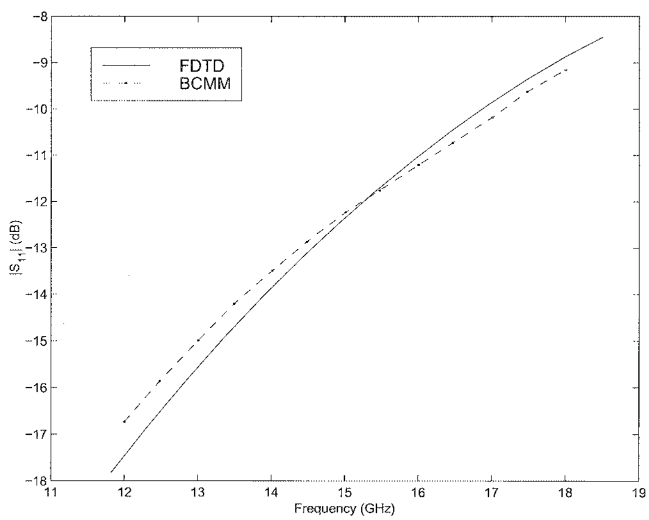

Fig. 4. Comparison of frequency domain results.

long as sufficient spectral energy is present at the frequencies of interest), because the spectral response eventually gets normalized. However, a correct choice of excitation source not only avoids any ambiguities as those reported in [9], but also allows one to terminate the FDTD simulation at earlier times since the fields may converge to zero much faster, as shown in Fig. 1.

IV. Numerical Verification

We use the equivalent electric field source to calculate the return loss of a mitered E-plane bend [11] for the same Ku-band waveguide considered before. The geometry is shown in Fig. 3. The uniform FDTD cell size is 0.2468 mm and anisotropic PML [12], [13] is used to terminate the waveguide.

The result is shown in Fig. 4 along with a boundary contour mode matching (BCMM) result [11]. Due to the fast convergence of the time domain response, the FDTD simulation can be terminated shortly after all reflected waves have passed through the observation plane. Fig. 4 shows that the FDTD result matched the BCMM result well. The residual error can be attributed to the staircase approximation of the diagonal portion of the bend, the residual reflections from the PML termination, and numerical dispersion.

V. Conclusion

In this letter, we have introduced a transparent electric field source for FDTD simulations of waveguide discontinuities which couples the same field (and spectrum) into the FDTD grid as a hard field source. This avoids the need to run an auxiliary simulation followed by time-domain convolutions. We have also provided a simple explanation for the "spectrum distortion" phenomenon reported in [9] in terms of the frequency dispersive characteristics and singular behavior of the waveguide impedance near the cut-off. Numerical experiments have been used to illustrate the main findings.

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